Int. J. Heat Mass Transfer. Vol. 15, pp. 359–360. Pergzmon Press 1972. Printed in Great Britain

ADDITIONAL DATA FOR CALCULATIONS OF STEADY HEAT OR MASS TRANSFER IN THE AXISYMMETRIC BOUNDARY LAYER OVER A CIRCULAR CYLINDER

L. VINCENT and F. COEURET

Centre de Cinétique Physique et Chimique du CNRS,
54-Villers. Nancy, France

(Received 7 August 1971)

NOMENCLATURE

k_{cylinder}, local mass (or heat) transfer coefficient for a cylinder;
$k_{\text{f.p.}}$, local mass (or heat) transfer coefficient for a flat plate;
M, fluid concentration or fluid temperature;
M_s, uniform wall concentration or temperature;
M_∞, fluid concentration or fluid temperature outside the layer;
Pr, Prandtl number;
r, distance from the axis of cylinder;
R, cylinder radius;
Sc, Schmidt number;
u, velocity component in x-direction;
U, uniform main stream velocity;
v, velocity component in r-direction;
x, distance along cylinder axis.

Greek symbols
α, diffusion coefficient or thermal conductivity;
η, $= (r - R)\cdot(U/\nu\cdot x)^{1/2}$;
θ, $= \dfrac{M - M_\infty}{M_s - M_\infty}$, dimensionless concentration or temperature;
$\theta_n(\eta)$, see equation (7);
$= [\partial\theta_n(\eta)/\partial\eta]_{\eta=0}$ with $n = 0, 1, 2, \ldots$;
ν, kinematic viscosity;
ξ, curvature parameter;
σ, Schmidt or Prandtl number.

THE PROBLEM of heat transfer through the axisymmetric laminar boundary layer over a circular cylinder maintained at a constant uniform temperature, has been solved by Bourne and Davies [1] and Eshghy and Hornbeck [2] for several values of the Prandtl number ($Pr = 0{\cdot}7$; 1; 10). Three solutions were obtained: one for small values of the downstream distance x, one for high values of x and an approximate solution joining the two others. Some additive notes have been published by these authors on the same subject [3–5].

During a work on mass transfer between cylinders and axial incompressible flows, using an electrochemical method for the determination of mass transfer coefficients [6], a similar theoretical problem had to be solved for high values of the Schmidt number.

This was solved for the particular case of small values of the curvature parameter ξ defined by:

$$\xi = \left(\frac{\nu\cdot x}{U\cdot R^2}\right)^{1/2} \tag{1}$$

using the method proposed by Eshghy and Hornbeck.

The boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{v}{r} + \frac{\partial v}{\partial r} = 0 \tag{2}$$

$$u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial r} = \frac{\nu}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial u}{\partial r}\right) \tag{3}$$

$$u\cdot\frac{\partial M}{\partial x} + v\cdot\frac{\partial M}{\partial r} = \frac{\nu}{\sigma\cdot r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial M}{\partial r}\right) \tag{4}$$

with the boundary conditions:

$$u(x, R) = v(x, R) = 0 \qquad u(x, \infty) = U \tag{5}$$

$$M(x, R) = M_s \qquad M(x, \infty) = M_\infty \tag{6}$$

Here M represents the concentration of the diffusing component (or the fluid temperature), and σ the Schmidt (or Prandtl) number, the others terms being defined in the list of symbols.

With this method, these equations are transformed into three sets of ordinary differential equations which have been presented by Eshghy and Hornbeck [2]; they have been solved numerically for several values of σ including that considered by Eshghy and Hornbeck. For this solution, a trial and error method based upon a Runge–Kutta algorithm has been used.

The results are given in the following table:

σ	$\theta_0'(0)$	$\theta_1'(0)$	$\theta_2'(0)$
0·7	−0·29268	−0·66364	0·62688
1	−0·33206	−0·69432	0·65658
10	−0·72814	−1·01761	1·12404
20	−0·925	−1·180	1·380
30	−1·0506	−1·2847	1·55885
100	−1·5718	−1·7206	2·302
300	−2·267	−2·3011	3·2946
500	−2·68	−2·652	3·8987
1000	−3·3870	−3·23875	4·9065
1500	−3·885	−3·655	5·625
2000	−4·2668	−3·9730	6·185
5000	−5·790	−5·2434	8·3733

where $\theta_0'(0)$, $\theta_1'(0)$ and $\theta_2'(0)$ have the same signification as in [2]. They are obtained by differenciation with $\eta = 0$ of the relative concentrations (or temperatures) $\theta_0(\eta)$, $\theta_1(\eta)$, $\theta_2(\eta)$ which appear in the series proposed to describe the concentration (or temperature) profiles in the boundary layer:

$$\theta = \theta_0(\eta) + \xi \cdot \theta_1(\eta) + \xi^2 \cdot \theta_2(\eta) + \cdots \quad (7)$$

These numerical results permit the calculation of the local mass (or heat) transfer coefficient to a cylinder, k_{cylinder}, for small values of the curvature parameter ξ, in an extended range of σ. This coefficient has the following expression:

$$k_{\text{cylinder}} = -\frac{\alpha}{R \cdot \xi}\{\theta_0'(0) + \xi \cdot \theta_1'(0) + \xi^2 \cdot \theta_2'(0) + \cdots\} \quad (8)$$

in which α symbolizes the diffusion coefficient of the diffusing component or the thermal conductivity of the fluid.

When $R \to \infty$, equation (8) gives the local mass (or heat) transfer coefficient for a laminar boundary layer over a flat plate:

$$k_{\text{f.p.}} = -\theta_0'(0) \cdot \alpha \cdot \left(\frac{U}{\nu x}\right)^{1/2}. \quad (9)$$

It can be seen that the tabulated values of $\theta_0'(0)$ agree very well with the asymptotic solution obtained by Cess [7]:

$$\theta_0'(0) = -0{\cdot}3388 \cdot \sigma^{1/3}. \quad (10)$$

REFERENCES

1. D. E. Bourne and D. R. Davies, Heat transfer through the laminar boundary layer on a circular cylinder in axial incompressible flow, *Q. J. Mech. Appl. Math.* **11**, 52–66 (1958).
2. S. Eshghy and R. W. Hornbeck, Flow and heat transfer in the axisymmetric boundary layer over a circular cylinder, *Int. J. Heat Mass Transfer* **10**, 1757–1766 (1967).
3. D. E. Bourne, D. R. Davies and S. Wardle, A further note on the calculation of heat transfer through the axisymmetrical laminar boundary layer on a circular cylinder, *Q. J. Mech. Appl. Math.* **12**, 257–260 (1959).
4. S. Eshghy and R. W. Hornbeck, Further remarks on axisymmetric boundary layer over a circular cylinder, *Int. J. Heat Mass Transfer* **12**, 243 (1969).
5. D. E. Bourne, Some remarks on calculations of heat transfer in the axisymmetric boundary layer over a circular cylinder, *Int. J. Heat Mass Transfer* **13**, 223 (1970).
6. F. Coeuret and F. Vergnes, Etude expérimentale du transfert de matière à un cylindre d'axe parallèle à un écoulement liquide, *C.R. Acad. Sci., Paris* **273C**, 580 (1971).
7. R. D. Cess, Magnetohydrodynamic effects upon heat transfer for laminar flow across a flat plate, *J. Heat Transfer* **82C**, 87–93 (1960).

Int. J. Heat Mass Transfer. Vol. 15, pp. 360–366. Pergamon Press 1972. Printed in Great Britain

USE OF THERMAL COMPARATOR METHOD FOR THERMAL CONDUCTIVITY MEASUREMENTS ON LIQUIDS: VALUES FOR THREE ORGANIC SERIES: NORMAL ALCOHOLS, ACIDS AND SATURATED HYDROCARBONS

R. W. POWELL and H. GROOT

Thermophysical Properties Research Center, Purdue University, West Lafayette, Indiana, U.S.A.

(*Received* 1 *June* 1971 *and in revised form* 28 *June* 1971)

NOMENCLATURE

C, degree Celsius;
K, Kelvin;
m, metre;
n, number of carbon atoms;
W, watt.

INTRODUCTION

Following the development by the senior author of